Journal of Engineering Physics and Thermophysics, Vol. 73, No. 3, 2000

PROBLEM CONCERNING THE INJECTION OF A SOLVENT INTO A POROUS MEDIUM SUBJECTED TO "SCLEROSIS"

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UDC 536.532

The problem on injecting a solvent into a porous medium clogged up because of the deposition of a solid phase (paraffins, bitumens) is considered. Within the framework of the schemes of plane-one-dimensional and radial-symmetric filtration, self-similar solutions are obtained that describe the distributions of density and velocity as well as evolution of a cleaned zone. Numerical evaluations are presented for a quantity which determines the laws of motion of the cleaned zone boundary as function of the parameters of the stratum and the injected solvent.

One of the most widespread reasons for worsening collector characteristics of the stratum in the well-bottom zone of oil wells are "sclerotic" changes because of the deposition of a solid phase (for example, paraffins) onto the walls of pore channels. In most cases, these depositions can be removed by injecting a solvent. Evaluations required to perform technological calculations using a solvent can be obtained from solutions of plane-one-dimensional and radial-symmetric problems. In particular, if the radial-symmetric formulation makes it possible to analyze the cleaning of a porous medium around the well, the plane-one-dimensional problem provides the possibility of tracing these processes near the racks (formed, for example, in hydraulic fracturing). Certain aspects of the displacement of a hydrocarbon liquid from porous media by means of solvents are considered in [1, 2].

Basic Equations. Suppose that a medium with porosity m in the initial state is partially clogged up by a solid phase which is dissolved in the injected liquid. In the initial state the volume fraction occupied by the solid phase is equal to ν, and therefore the "living" porosity is $m' = (1-\nu)m$. Moreover, the clogged porous medium in turn is saturated with the liquid. In the injection of a solvent into this system one can single out three characteristic zones: a near porous medium cleaned from the solid phase (with porosity m) where a pure solvent is present in pores, an intermediate zone (with porosity m') in which the solvent, saturated with the solid phase, is filtered, and a distant zone where the filtration flow of the initial saturating liquid occurs. It should be noted that according to the concepts adopted, these zones contain three dissimilar liquids that differ in viscosity and equilibrium values of density. We will assume that filtration processes during the solvent injection occur under elastic conditions. Then the linear equation of piezoconductivity and Darcy's law can be written in the form

$$\frac{\partial p_i}{\partial t} = \chi_i \frac{1}{r^n} \frac{\partial}{\partial r}\left(r^n \frac{\partial p_i}{\partial r} \right), \quad u_i = m_i v_i = -\frac{k_i}{\mu_i} \frac{\partial p_i}{\partial r}, \tag{1}$$

$$\chi_i = \frac{k_i}{\mu_i \beta_i}, \quad \beta_i = m_i \beta_{\mathrm{liq}\, i} + \beta_{\mathrm{s}i},$$

$$m_1 = m, \quad m_2 = m_3 = m' = m(1-\nu), \quad k_2 = k_3 .$$

Sterlitamak State Pedagogical Institute, Sterlitamak, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 73, No. 3, pp. 520-528, May–June, 2000. Original article submitted January 12, 1999.

Here $n = 0$ and 1 correspond to the plane-one-dimensional and radial-symmetric problems.

The assumptions adopted above for the structure of the zones actually ignore the extension of the regions in which the solid phase is dissolved and washed off. Thereby these regions are replaced by fracture surfaces for a part of the variables (for example, the filtration rate) and parameters that determine the filtration characteristics (porosity, permeability, and viscosity). Moreover, hereafter we will ignore the hydraulic resistance in these regions, and at the boundaries between the zones we will require the fulfillment of the condition for continuity of pressure:

$$p_1 = p_2 = p_{(12)}\ (r = r_{(12)})\ ,\quad p_2 = p_3 = p_{(23)}\ (r = r_{(23)})\ .$$

As a whole, from the mass conservation law for the entire solvent–solid phase system at the boundary between the first and second zones it follows that

$$\rho_1 m_1 \left(v_1 - \frac{dr_{(12)}}{dt} \right) = \rho_2 m_2 \left(v_2 - \frac{dr_{(12)}}{dt} \right) - \rho_s m \nu \frac{dr_{(12)}}{dt} \quad (r = r_{(12)})\ , \tag{2}$$

where ρ_s is the solid-phase density. We also write the equation of mass conservation of the solvent at this boundary as

$$\rho_1 m_1 \left(v_1 - \frac{dr_{(12)}}{dt} \right) = (1 - g)\, \rho_2 m_2 \left(v_2 - \frac{dr_{(12)}}{dt} \right) \quad (r = r_{(12)})\ ; \tag{3}$$

here g is the mass concentration of the solid phase in the solvent in the saturation state. Formulas (2) and (3) are written in a linearized approximation, and their equilibrium values are taken for the liquid densities. We note also that allowance for the change in the densities due to the pressure increase compared to the equilibrium value will introduce an error of the order of $\Delta\tilde{\rho}_i << 1$ ($\Delta\tilde{\rho}_i = \Delta\rho_i/\rho_i$, $\Delta\rho_i$ is the maximum change in the density because of compression). Relations (2) and (3) with account for Darcy's law from Eq. (1) can be represented in the form

$$\rho_1 \frac{k_1}{\mu_1} \frac{\partial p_1}{\partial r} - \rho_2 \frac{k_2}{\mu_2} \frac{\partial p_2}{\partial r} = m\, (\rho_2 (1 - \nu) + \rho_s \nu - \rho_1) \frac{dr_{(12)}}{dt}\ ,$$

$$\rho_1 \frac{k_1}{\mu_1} \frac{\partial p_1}{\partial r} - (1 - g)\, \rho_2 \frac{k_2}{\mu_2} \frac{\partial p_2}{\partial r} = m\, ((1 - g)\, \rho_2 (1 - \nu) - \rho_1) \frac{dr_{(12)}}{dt} \quad (r = r_{(12)})\ .$$

In what follows, for convenience, these expressions will be transformed in the following manner:

$$\begin{aligned} \frac{k_1}{\mu_1} \frac{\partial p_1}{\partial r} &= -m \frac{(1 - g)\, \rho_s \nu + g \rho_1}{g \rho_1} \frac{dr_{(12)}}{dt}\ , \\ \frac{k_2}{\mu_2} \frac{\partial p_2}{\partial r} &= -m \frac{g \rho_2 (1 - \nu)\ + \rho_s \nu}{g \rho_2} \frac{dr_{(12)}}{dt} \quad (r = r_{(12)})\ . \end{aligned} \tag{4}$$

We assume that at the boundary between the second and third zones there is the condition for immiscible displacement, from which it is seen that this boundary is also the surface of contact fracture. Then we write

$$\rho_2 m_2 \left(v_2 - \frac{dr_{(23)}}{dt} \right) = \rho_3 m_3 \left(v_3 - \frac{dr_{(12)}}{dt} \right) = 0 \quad (r = r_{(23)})\ .$$

Hence with account for Darcy's law from Eq. (1), we have

$$\frac{k_2}{\mu_2}\frac{\partial p_2}{\partial r}=\frac{k_3}{\mu_3}\frac{\partial p_3}{\partial r}=-m\,(1-\nu)\frac{dr_{(23)}}{dt}\quad (r=r_{(23)})\,. \tag{5}$$

In the case of the absence of solid depositions in the initial state ($\nu = 0$), it follows that the intermediate region will be absent ($r_{(12)} = r_{(23)}$), and then we obtain the well-known results [3].

If, for the dependences of the coefficients of absolute permeability on the "living" porosity, we take the Kozeny–von Kármán formulas, then it is possible to write

$$k_1=k_0\frac{m^3}{(1-m)^2}\,,\quad k_2=k_3=k_0\frac{m^3\,(1-\nu)^3}{(1-m\,(1-\nu))^2}\,; \tag{6}$$

here k_0 is the parameter responsible for the characteristic dimensions of the pores.

Plane-One-Dimensional Problem ($n = 0$, $r = x$). Suppose that the solvent injection occurs on sudden increase in the pressure from the initial value in the porous medium p_0 to a certain constant value p_e at its boundary ($p_e > p_0$):

$$p_3=p_0\quad (x>0\,,\ t=0)\,,\quad p_1=p_e\quad (x=0\,,\ t>0)\,. \tag{7}$$

This problem has the following self-similar solution:

$$p_1=p_e+(p_{(12)}-p_e)\frac{\int\limits_0^{\xi}\exp\left(-\frac{\xi'^2}{4}\right)d\xi'}{\int\limits_0^{\xi_{(12)}}\exp\left(-\frac{\xi'^2}{4}\right)d\xi'}\qquad (0<\xi<\xi_{(12)})\,,$$

$$p_2=p_{(12)}+(p_{(23)}-p_{(12)})\frac{\int\limits_{\xi_{(12)}}^{\xi}\exp\left(-\frac{\xi'^2}{4\eta_2}\right)d\xi'}{\int\limits_{\xi_{(12)}}^{\xi_{(23)}}\exp\left(-\frac{\xi'^2}{4\eta_2}\right)d\xi'}\qquad (\xi_{(12)}<\xi<\xi_{(23)})\,,$$

$$p_3=p_{(23)}+(p_0-p_{(23)})\frac{\int\limits_{\xi_{(23)}}^{\xi}\exp\left(-\frac{\xi'^2}{4\eta_3}\right)d\xi'}{\int\limits_{\xi_{(23)}}^{\infty}\exp\left(-\frac{\xi'^2}{4\eta_3}\right)d\xi'}\qquad (\xi_{(23)}<\xi<\infty)\,,$$

$$\xi=x/\sqrt{\chi_1 t}\,,\quad \eta_i=\chi_i/\chi_1\quad (i=2,3)\,.$$

Using this solution and boundary conditions (4) and (5), we can write the following system of transcendental equations for determining in self-similar variables the coordinates of the boundaries $\xi_{(12)}$ and $\xi_{(23)}$ between the zones and the values of the pressures $p_{(12)}$ and $p_{(23)}$ at these boundaries:

$$\frac{k_1}{\mu_1}\frac{(p_{(12)}-p_e)\exp\left(-\xi_{(12)}^2/4\right)}{\int\limits_0^{\xi_{(12)}}\exp\left(-\frac{\xi'^2}{4}\right)d\xi'}=-m\frac{(1-g)\,\rho_s\nu+g\rho_1}{g\rho_1}\chi_1\frac{\xi_{(12)}}{2}\,,$$

$$\frac{k_2}{\mu_2}\frac{(p_{(23)}-p_{(12)})\exp(-\xi_{(12)}^2/4\eta_2)}{\int\limits_{\xi_{(12)}}^{\xi_{(23)}}\exp\left(-\frac{\xi'^2}{4\eta_2}\right)d\xi'}=-m\frac{g\rho_2(1-\nu)+\rho_s\nu}{g\rho_2}\chi_1\frac{\xi_{(12)}}{2},$$

$$\frac{k_2}{\mu_2}\frac{(p_{(23)}-p_{(12)})\exp(-\xi_{(23)}^2/4\eta_2)}{\int\limits_{\xi_{(12)}}^{\xi_{(23)}}\exp\left(-\frac{\xi'^2}{4\eta_2}\right)d\xi'}=\frac{k_3}{\mu_3}\frac{(p_0-p_{(23)})\exp(-\xi_{(23)}^2/4\eta_3)}{\int\limits_{\xi_{(23)}}^{\infty}\exp\left(-\frac{\xi'^2}{4\eta_3}\right)d\xi'}=$$

$$=-m(1-\nu)\chi_1\frac{\xi_{(23)}}{2}.$$

From this system it is easy to obtain two equations for determination of $\xi_{(12)}$ and $\xi_{(23)}$ as functions of the pressure drop $\Delta p(\Delta p = p_e - p_0)$:

$$g\exp[(\xi_{(23)}^2-\xi_{(12)}^2)/4\eta_2]=\frac{g\rho_2(1-\nu)+\rho_s\nu}{\rho_2(1-\nu)}\frac{\xi_{(12)}}{\xi_{(23)}},$$

$$\frac{(1-g)\rho_s\nu+g\rho_1}{g\rho_1}\frac{\mu_1}{k_1}\xi_{(12)}\exp\left(\frac{\xi_{(12)}^2}{4}\right)\int\limits_0^{\xi_{(12)}}\exp\left(-\frac{\xi'^2}{4}\right)d\xi'-$$

$$-(1-\nu)\xi_{(23)}\frac{\mu_2}{k_2}\exp\left(\frac{\xi_{(23)}^2}{4\eta_2}\right)\int\limits_{\xi_{(12)}}^{\xi_{(23)}}\exp\left(-\frac{\xi'^2}{4\eta_2}\right)d\xi'-$$

$$-(1-\nu)\xi_{(23)}\frac{\mu_3}{k_3}\exp\left(\frac{\xi_{(23)}^2}{4\eta_3}\right)\int\limits_{\xi_{(23)}}^{\infty}\exp\left(-\frac{\xi'^2}{4\eta_3}\right)d\xi'=-2\frac{\Delta p}{m\chi_1}. \quad (8)$$

Here the pressures at the boundaries between the zones can be found from the following expressions:

$$p_{(12)}=p_e-\frac{1}{2}\frac{\mu_1}{k_1}\xi_{(12)}\chi_1 m\frac{(1-g)\rho_s\nu+g\rho_1}{g\rho_1}\exp\left(\frac{\xi_{(12)}^2}{4}\right)\int\limits_0^{\xi_{(12)}}\exp\left(-\frac{\xi'^2}{4}\right)d\xi',$$

$$p_{(23)}=p_0+\frac{1}{2}\frac{\mu_3}{k_3}m(1-\nu)\chi_1\xi_{(23)}\exp\left(\frac{\xi_{(23)}^2}{4\eta_3}\right)\int\limits_{\xi_{(23)}}^{\infty}\exp\left(-\frac{\xi'^2}{4\eta_3}\right)d\xi'.$$

On the basis of the solutions obtained, we carried out numerical calculations for a system in which kerosene is a solvent and bitumen is a soluble solid phase. For this system at temperatures T = 291 and 343 K using the Kendall formula and the data of [4], we have the following values of the parameters: g = 0.6 and 0.9, μ_1 = 0.00137 and 0.0007 Pa·sec, μ_2 = 0.00745 and 0.00093 Pa·sec, μ_3 = 0.09391 and 0.011 Pa·sec, $\rho_1 \approx \rho_2 \approx 900$ kg/m^3, and $\rho_3 \approx 700$ kg/m^3. For the characteristics of the porous medium we take values of the parameters that are equal to m = 0.3, $k_1 = 10^{-12}$ m^2, and ν = 0.5. Then, according to Eqs. (1) and (6), we have $m_2 = m_3 = 0.15$ and $k_2 = k_3 = 8.48\cdot10^{-14}$ m^2. Using the above values of the parameters of the liquid and the

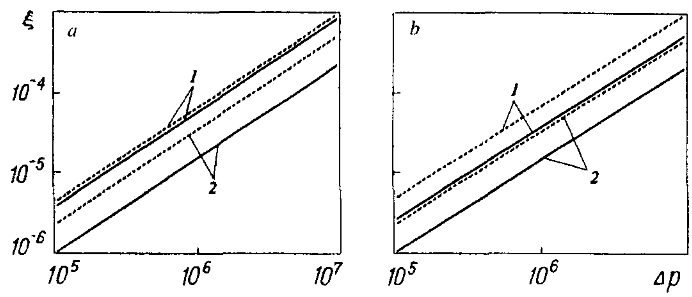

Fig. 1. Dependences of the parameters $\xi_{(12)}$ (solid curve) and $\xi_{(23)}$ (dashed curve) on the pressure drop under the influence of the degree of clogging (a) and the solvent (b) on the evolution of motion of the cleaned zone boundary (a, 1 and 2 correspond to ν = 0.1 and 0.5; b, 1 and 2 for g = 0.9 and 0.6, ν = 0.5). Δp, Pa.

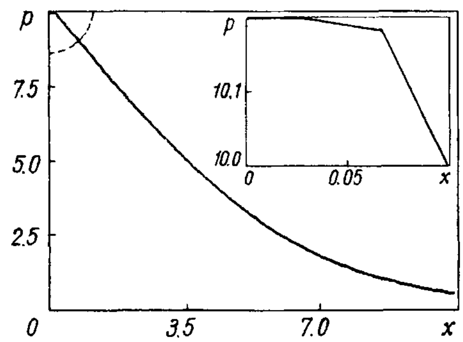

Fig. 2. Distribution of the pressure near the boundary of the porous medium through which the solvent is injected (the inset illustrates the pressure distribution in the first and second zones in magnified form; the parameters of the stratum and the solvent are the same as for Fig. 1). p, MPa, x, m.

porous medium in different zones, for the coefficients of piezoconductivity we obtain $\chi_1 \approx 3.8$ and 7.45 m^2/sec, $\chi_2 \approx 0.06$ and 0.45 m^2/sec, and $\chi_3 \approx 0.004$ and 0.032 m^2/sec.

Figure 1 presents the dependences of the dimensionless parameters $\xi_{(12)}$ and $\xi_{(23)}$ that determine the boundaries of the cleaned zone and the displaced liquid on the pressure drop $\Delta p(\Delta p = p_e - p_0)$ at different values of the volume fraction, occupied by the solid phase (ν = 0.1 and 0.5), and the equilibrium mass concentration of the solid phase in the solvent (g = 0.6 and 0.9). As follows from the figures, the boundary of the cleaned zone moves more quickly, the less the stratum is clogged up and the "stronger" the solvent.

Figure 2 illustrates the pressure profile 1 h later after the onset of the solvent injection (g = 0.6) into the stratum (ν = 0.5). It is seen that the basic pressure drop occurs in the uncleaned zone of the stratum.

In most cases the following evaluations are observed:

$$\frac{\rho_s}{\rho_1} \sim 1\,, \quad \frac{\rho_2}{\rho_1} \sim 1\,, \quad \frac{\rho_3}{\rho_1} \sim 1\,, \quad \frac{g\,(1-\nu)}{g\,(1-\nu)+\nu} \sim 1\,,$$

$$\frac{\mu_2}{\mu_1} \sim 1\,, \quad \frac{\mu_3}{\mu_1} \sim 1\,, \quad \frac{k_2}{k_1} \sim 1\,, \quad \frac{k_3}{k_1} \sim 1\,, \quad \eta_2 \sim 1\,, \quad \eta_3 \sim 1\,.$$

Then for the solutions satisfying the conditions

$$\xi_{(12)} << 1\,, \quad \xi_{(23)} << 1\,, \tag{9}$$

we obtain

$$\xi_{(12)} = \frac{g\rho_2 (1-\nu)}{g\rho_2 (1-\nu) + \rho_s \nu} \xi_{(23)} , \quad (1-\nu) \frac{\mu_3}{k_3} \sqrt{\eta_3 \pi}\, \xi_{(23)} = 2 \frac{\Delta p}{m\chi_1} . \tag{10}$$

The second equation of (10) yields

$$\xi_{(23)} = 2 \frac{\Delta p}{m} \frac{k_3}{\mu_3} \frac{1}{(1-\nu)} \sqrt{\left(\frac{1}{\chi_3 \chi_1 \pi} \right)} . \tag{11}$$

From this one can see that solutions (10) and (11) for $\xi_{(12)}$ and $\xi_{(23)}$ satisfy conditions (9) at a rather weak pressure drop:

$$\Delta p << \sqrt{1-\nu}\, \chi_1 \frac{\mu_1}{k_1} m .$$

In this case, in the first and second zones we have the quasistationary profiles of the pressure distribution that are determined by the following formulas:

$$p_1 = p_e + (p_{(12)} - p_e) \frac{\xi}{\xi_{(12)}} \quad (0 < \xi < \xi_{(12)}) ,$$

$$p_2 = p_{(12)} + (p_{(23)} - p_{(12)}) \frac{\xi - \xi_{(12)}}{\xi_{(23)} - \xi_{(12)}} \quad (\xi_{(12)} < \xi < \xi_{(23)}) ,$$

while for the values of the pressures at the boundaries between the zones we have

$$p_{(12)} = p_e - \frac{1}{2} \frac{\mu_1}{k_1} \chi_1 m \frac{(1-g)\rho_s \nu + g\rho_1}{g\rho_1} \xi_{(12)}^2 , \tag{12}$$

$$p_{(23)} = p_e - \frac{1}{2} \chi_1 \frac{\mu_1}{k_1} m \left[\frac{(1-g)\rho_s \nu + g\rho_1}{g\rho_1} + \frac{\mu_2}{\mu_1} \frac{k_1}{k_2} \frac{g\rho_2 (1-\nu) + \rho_s \nu}{g\rho_2} \frac{\rho_s \nu}{g\rho_2 (1-\nu)} \right] \xi_{(12)}^2 .$$

It should be noted here that the simple formulas (10)-(12) obtained for the basic characteristics of the first (cleaned) zone in the majority of cases (from the viewpoint of practical applications) hold true valid in a rather wide range of parameters for the initial porous medium and the solvent, and also for the pressure drop Δp.

Radial-Symmetric Problem (n = 1). From a certain instant of time, the problem for the case of solvent injection with a constant volumetric flow rate per unit length of the well also has the self-similar solution. Here the corresponding initial and boundary conditions will be written in the form

$$p_3 = p_0 \quad (t = 0 , \ r > 0) ,$$

$$2\pi \frac{k_1}{\mu_1} r_w \left(\frac{\partial p}{\partial r} \right)_{r_w} = q \quad (t > 0 , \ r_w \to 0) .$$

The solution similar to (7) appears as

$$p_1 = p_{(12)} + \hat{q} \int_{\xi}^{\xi_{(12)}} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4} \right) d\xi' \quad (0 < \xi < \xi_{(12)}) ,$$

$$p_2 = p_{(12)} + (p_{(23)} - p_{(12)}) \frac{\int\limits_{\xi_{(12)}}^{\xi} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_2}\right) d\xi'}{\int\limits_{\xi_{(12)}}^{\xi_{(23)}} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_2}\right) d\xi'} \quad (\xi_{(12)} < \xi < \xi_{(23)}) , \tag{13}$$

$$p_3 = p_{(23)} - \frac{(p_{(23)} - p_0) \int\limits_{\xi_{(23)}}^{\xi} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_3}\right) d\xi'}{\int\limits_{\xi_{(23)}}^{\infty} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_3}\right) d\xi'} \quad (\xi_{(23)} < \xi < \infty) ,$$

$$\hat{q} = \frac{q}{2\pi} \frac{\mu_1}{k_1} .$$

Using boundary conditions (4) and Eqs. (13), we write

$$\frac{k_1}{\mu_1} \hat{q} \exp\left(-\frac{\xi_{(12)}^2}{4}\right) = m \frac{(1-g)\,\rho_s \nu + g\rho_1}{g\rho_1} \chi_1 \frac{\xi_{(12)}^2}{2} ,$$

$$\frac{k_2}{\mu_2} \frac{(p_{(23)} - p_{(12)}) \exp\left(-\frac{\xi'_{(12)}}{4\eta_2}\right)}{\int\limits_{\xi_{(12)}}^{\xi_{(23)}} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_2}\right) d\xi'} = -m \frac{g\rho_2 (1-\nu) + \rho_s \nu}{g\rho_2} \chi_1 \frac{\xi_{(12)}^2}{2} , \tag{14}$$

$$\frac{k_2}{\mu_2} \frac{(p_{(23)} - p_{(12)}) \exp\left(-\frac{\xi_{(23)}^2}{4\eta_2}\right)}{\int\limits_{\xi_{(12)}}^{\xi_{(23)}} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_2}\right) d\xi'} = \frac{k_3}{\mu_3} \frac{(p_0 - p_{(23)}) \exp\left(-\frac{\xi_{(23)}^2}{4\eta_3}\right)}{\int\limits_{\xi_{(23)}}^{\infty} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_3}\right) d\xi'} = -m (1-\nu) \chi_1 \frac{\xi_{(23)}^2}{2} .$$

Hence for determining $\xi_{(12)}$ and $\xi(23)$ we have

$$g \exp\left(\frac{\xi_{(23)}^2 - \xi_{(12)}^2}{4\eta_2}\right) = \frac{g\rho_2 (1-\nu) + \rho_s \nu}{\rho_2 (1-\nu)} \left(\frac{\xi_{(12)}}{\xi_{(23)}}\right)^2 ,$$

$$\xi_{(12)}^2 \exp\left(\frac{\xi_{(12)}^2}{4}\right) = 2\rho_1 \frac{\hat{q} g}{m\,((1-g)\,\rho_s \nu + g\rho_1)} \frac{k_1}{\mu_1} . \tag{15}$$

By means of the last equation from system (14) we find the pressures between the zones:

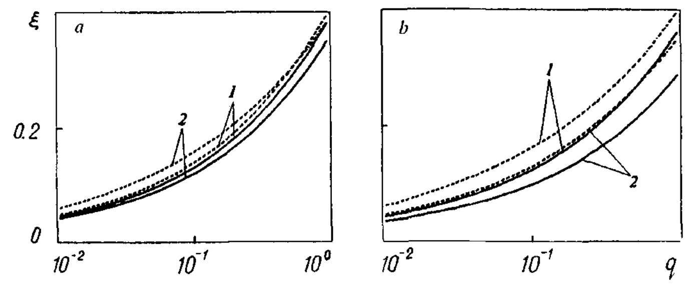

Fig. 3. Dependences of the boundaries of the cleaned zone (solid curve) and the displaced liquid (dashed curve) on the flow rate of the injected solvent with different initial clogging-up (determined by ν) of the porous medium (a) and on the quality of the solvent (determined by g) (b) (the numbering of the curves is the same as in Fig. 1). q, m^3/(m·sec).

$$p_{(23)} = p_0 + (1-\nu)\, m\chi_1 \frac{\mu_3}{k_3} \frac{\xi_{(23)}^2}{2} \exp\left(\frac{\xi_{(23)}^2}{4\eta_3}\right) \int\limits_{\xi_{(23)}}^{\infty} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_3}\right) d\xi',$$

$$p_{(12)} = p_{(23)} + m\chi_1 (1-\nu) \frac{\mu_2}{k_2} \frac{\xi_{(23)}^2}{2} \exp\left(\frac{\xi_{(23)}^2}{4\eta_2}\right) \int\limits_{\xi_{(12)}}^{\xi_{(23)}} \xi'^{-1} \exp\left(-\frac{\xi'^2}{4\eta_2}\right) d\xi'. \qquad (16)$$

Figure 3 presents the calculation results for the dependences of the dimensionless boundaries of the cleaned zone and the displaced liquid on the volumetric flow rate of the injected solvent with different initial clogging-up of the stratum (a) as well as on the resolving power of the liquid (b) injected into the stratum. It can be seen that the weaker the stratum is clogged up, the smaller the distance between the boundaries of the two zones.

For the roots of Eqs. (15) satisfying the conditions $\xi_{(12)} << 1$ and $\xi_{(23)} << 1$, we have

$$\xi_{(12)} = \sqrt{\left(2\frac{k_1}{\mu_1} \frac{g\,\hat{q}\,\rho_1}{((1-g)\,\rho_s\nu + g\rho_2)\, m}\right)}, \quad \xi_{(23)} = \xi_{(12)} \sqrt{\left(\frac{g\rho_2 (1-\nu) + \rho_s\nu}{g\rho_2 (1-\nu)}\right)}. \qquad (17)$$

Using the asymptotics for the integral exponential function at small values of the argument ($x << 1$)

$$-\,\mathrm{Ei}\,(-x) \approx \ln\frac{1}{x} - 0.5772\,,$$

from expressions (16) for the pressures between the zones we can obtain

$$p_{(23)} = p_0 + m\,(1-\nu) \frac{\mu_3}{k_3} \chi_1 \frac{\xi_{(23)}^2}{4} \left(\ln\frac{4\eta_3}{\xi_{(23)}^2} - 0.5772\right),$$

$$p_{(12)} = p_{(23)} + (1-\nu) \frac{\mu_2}{k_2} \chi_1 m \frac{\xi_{(23)}^2}{2} \left(\ln\frac{\xi_{(23)}}{\xi_{(12)}^2}\right).$$

Here for the pressure distribution we have

$$p_1 = p_{(12)} + 2\hat{q} \ln\left(\frac{\xi_{(12)}}{\xi}\right) \quad (0 < \xi < \xi_{(12)})\,,$$

$$p_2 = p_{(12)} + (p_{(23)} - p_{(12)}) \frac{\ln\left(\frac{\xi_{(12)}}{\xi}\right)}{\ln\left(\frac{\xi_{(12)}}{\xi_{(23)}}\right)} \quad (\xi_{(12)} < \xi < \xi_{(23)}) ,$$

$$p_3 = p_{(23)} + (p_0 - p_{(23)}) \frac{\ln\left(\frac{\xi_{(23)}}{\xi}\right)}{\ln\left(\frac{4\eta_3}{\xi_{(23)}} - 0.5772\right)} \quad (\xi_{(23)} < \xi , \quad \xi << 1) .$$

These solutions hold true at rather weak flow rates of injection that satisfy the condition

$$q << q_* , \quad q_* = \frac{\pi}{\rho_1} \frac{((1 - g) \rho_s \nu + g\rho_2) \chi_1 m}{g} .$$

NOTATION

p_i, pressure in the ith zone, Pa; $p_{(12)}$, pressure at the boundary of the first and second zones, Pa; $p_{(23)}$, pressure at the boundary of the second and third zones, Pa; T, temperature, K; t, time, h; r, distance, m; r_w, radius of the well; $r_{(12)}$, boundary of the first zone, m; $r_{(23)}$, boundary of the second zone, m; ν, volume fraction occupied by the solid phase; v_i, true velocity, m/sec; u_i, filtration rate, m/sec; m_i, coefficient of porosity in the ith zone; k_i, coefficient of permeability in the ith zone, m^2; $\beta_{\text{liq}i}$ and β_{si}, coefficients of elastic capacity of the liquid and the stratum in the ith zone, Pa^{-1}; μ_i, dynamic viscosity, Pa·sec; χ_i, coefficient of piezoconductivity, m^2/sec; ρ_1, density of the solvent, kg/m^3; ρ_2, density of the mixture, kg/m^3; $\xi_{(12)}$ and $\xi_{(23)}$, dimensionless self-similar variables; q and q_*, flow rate and critical flow rate of the solvent per unit length of the well, m^3/(sec·m). Subscripts: i = 1, 2, 3, values of the parameters corresponding to the 1st, 2nd, and 3rd zones; liq$i$, liquid in the ith zone; si, porous medium where filtration flow occurs; s, skeleton; 0, initial value; e, equilibrium state; w, well.

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